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Magnetic Flux in Toroidal Type I Compactification

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Abstract: We discuss the compactification of type I strings on a torus with additional background gauge flux on the D9-branes. The solutions to the cancellation of the RR tadpoles display various phenomenologically attractive features: supersymmetry breaking, chiral fermions and the opportunity to reduce the rank of the gauge group as desired. We also point out the equivalence of the concept of various different background fields and noncommutative deformations of the geometry on the individual D9-branes by constructing the relevant boundary states to describe such objects.

1 Introduction

It is of conceptual interest on its own behalf to study the effects of nontrivial background gauge fields in type I string theory [1, 2, 3, 4]. In particular the presence of constant magnetic flux on a torus still allows a microscopic description of the relevant degrees of freedom in terms of the worldsheet theory. A very prominent feature is that a constant flux induces a deformation of the background geometry from a commutative towards a noncommutative coordinate algebra [5, 6]. More precisely, we discuss the presence of various different fluxes on different D9-branes, which gives rise to noncommutative deformations of the coordinate algebras of open string coordinates on the D9-branes, the deformation parameter governed by the respective value of the flux. To prove the one-loop consistency of such compactifications we show that they still allow to cancel the RR tadpole of the Klein bottle. Interestingly, D9-branes with nonvanishing flux also couple to tensor fields of degree lower than ten, they carry D5-brane charge as well.

We find attractive phenomenological features for the effective theory of type I compactified on a four- or sixdimensional torus with appropriate flux on the D9-branes. The spectrum includes chiral fermions. The gauge group can be engineered to be any product of unitary, orthogonal and symplectic groups with the upper bound of 16 for the rank. Supersymmetry is always broken, which implies that the one-loop amplitude is nonvanishing. A NSNS tadpole is left over and open string tachyons appear, leaving the dynamical stability of the configuration an open question.

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The paper is organized as follows. In section 2 we first discuss D-branes on a torus with an additional magnetic background field $\mathcal{F}$ on their world volume and its equivalence to a noncommutative deformation of their open string coordinate algebra. We also show how a better intuition of such D-brane set-ups can be obtained by employing a T-dual version with D-branes at angles. In section 3 we then compute the tadpole cancellation conditions for type I strings, when the D9-branes carry such magnetic flux and discuss the implications of the solutions. Finally we present a semi-realistic example and point out two major obstructions to obtain models with concrete phenomenological impact.

2 D-branes with $\mathcal{F}$-Flux, Non-commutativity and T-duality

We compactify type I string theory on a six- or fourdimensional torus, adding magnetic background flux on the D-branes. Therefore we first perform a preliminary analysis of D-branes wrapping a simple twodimensional torus $\mathbb{T}^2$, which carries a nontrivial $U(1)$ gauge field with constant field strength on its world volume. The $\mathbb{T}^2$ is chosen to have purely imaginary complex and Kähler structures for simplicity. They define the spectrum of bosonic zero-modes, KK momenta and winding states. The constant background flux is given by the $U(1)$ gauge curvature $\mathcal{F}_{ij} = \mathcal{F}\delta_{ij}$. Physically inequivalent background fields are classified by their first Chern number $m \in \mathbb{Z} \simeq H^2(\mathbb{T}^2, \mathbb{Z})$, which is the magnetic charge of the field configuration. The possible values for $\mathcal{F}$ are given by

$$\mathcal{F} = \frac{2\pi m}{nR_1R_2}, \tag{1}$$

where $n \in \mathbb{Z}$ denotes the electric charge quantum of the particular $U(1)$. Thus, any stack of coincident D$_\mu$-branes wrapping the torus is charcterized by the two integers n_μ and m_μ, the electric and magnetic charge quanta of its worldvolume gauge theory.

The D-branes themselves are described in CFT by a corresponding boundary state defined by the boundary conditions with background $\mathcal{F}$-flux

$$\begin{aligned} (\partial_\sigma X_1 + \mathcal{F}_\mu\, \partial_\tau X_2)\,|_{\partial\Sigma_\mu} &= 0, \\ (\partial_\sigma X_2 - \mathcal{F}_\mu\, \partial_\tau X_1)\,|_{\partial\Sigma_\mu} &= 0. \end{aligned} \tag{2}$$

The mode expansion of the coordinate fields of a string stretching between two D-branes μ and ν then displays Fourier modings in $\mathbb{Z} + (\phi_\mu - \phi_\nu)/\pi$, where we defined $\phi_\mu \equiv \operatorname{arccot}(\mathcal{F}_\mu)$. It is by now well known and widely appreciated that coordinates of open strings ending on D-branes with magnetic flux on their world volume do not commute. The relevant "open string metric" and "open string antisymmetric tensor field" are [6]

$$G_\mu^{ij} \equiv \frac{1}{1+\mathcal{F}_\mu^2}\delta^{ij}, \quad \theta_\mu^{ij} \equiv -\frac{2\pi\mathcal{F}_\mu}{1+\mathcal{F}_\mu^2}\epsilon^{ij}. \tag{3}$$

The commutator of the open string coordinates can then be expressed in terms of the deformation parameter θ_μ^{12}

$$[X_1(\tau,\sigma), X_2(\tau,\sigma')]\,|_{\sigma=\sigma'\in\partial\Sigma_\mu} = -\frac{2\pi\mathcal{F}_\mu}{1+\mathcal{F}_\mu^2} = i\theta_\mu^{12}. \tag{4}$$

The boundary state $|\mathrm{B}_\mu\rangle$ that solves the boundary conditions

$$\begin{aligned} (p_\mathrm{L} + p_\mathrm{R} - i\mathcal{F}_\mu\,(p_\mathrm{L} - p_\mathrm{R}))\,|\mathrm{B}_\mu\rangle &= 0, \\ (\alpha_q + \exp(2i\phi_\mu)\,\tilde{\alpha}_{-q})\,|\mathrm{B}_\mu\rangle &= 0 \end{aligned} \tag{5}$$

is represented by the coherent state (here the bosonic part)

$$|\mathrm{B}_\mu\rangle = \exp\left(\sum_{q>0} \frac{1}{q} e^{2i\phi_\mu} \alpha_{-q}\tilde{\alpha}_{-q} + \text{ c.c.}\right) \sum_{r^1,r^2\in\mathbb{Z}} |r^1, r^2\rangle_{(\mathcal{F}_\mu)}. \tag{6}$$

The zero mode spectrum can be explicitly evaluated and it turns out that it can be summarized by using (3) as well:

$$p^i_\mu = \frac{1}{1+\mathcal{F}^2_\mu} \frac{r^i}{n_\mu R_i} = G^{ij}_\mu \frac{r^j}{n_\mu R_j}, \quad w^i_\mu = \frac{i\mathcal{F}_\mu \epsilon^{ij}}{1+\mathcal{F}^2_\mu} \frac{r^j}{n_\mu R_j} = \frac{\theta^{ij}_\mu}{2\pi} \frac{r^j}{n_\mu R_j} \tag{7}$$

giving rise to the open string mass spectrum

$$M^2_{\mu,\text{open}} = \frac{r^i}{n_\mu R_i} G^{ij}_\mu \frac{r^j}{n_\mu R_j}. \tag{8}$$

We recognize the electric charge quantum n_μ entering as a winding number for the D_μ-brane.

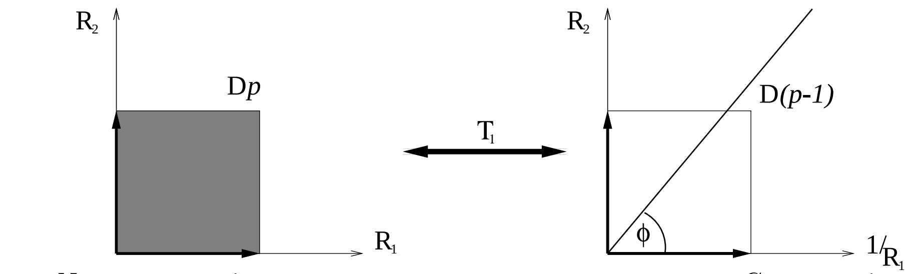
Summarizing, the internal space of type I string theory on a torus with extra magnetic fluxes on the D9-branes is noncommutative, while the noncompact gauge theory stays commutative. Furthermore, it is strictly equivalent to employ the open string metric and antisymmetric tensor in all computations or to explicitly implement the background gauge field into the boundary conditions.

There is another equivalence which we only note as an aside. The deformation induced by the background gauge field is again identical to performing a left-right asymmetric rotation of the closed string coordinates [7]. This establishes an interesting link between asymmetric string vacua and noncommutative geometry. Asymmetric nongeometric rotational symmetries are gauged in asymmetric orbifolds or orientifolds, such that these spaces do not distinguish between certain values of the noncommutativity parameters θ^{ij}. In [7] a large class of asymmetric orientifolds with these properties has been constructed, which are related to a previously explored type of symmetric and therefore geometric orientifolds with D-branes at angels [8, 9, 10, 11] via a certain T-duality, which we now come to discuss.

It is very helpful for the visualisation of D-branes with various fluxes to perform a T-duality in one of the directions of the $\mathbb{T}^2$, say x_1. The complex and Kähler structures get exchanged and the boundary conditions of open strings on Dp-branes with $\mathcal{F}_\mu$-flux switch to D$(p-1)$-branes at an angle ϕ_μ relative to the x_1 axis:

$$\begin{aligned} \partial_\sigma X_1 + \mathcal{F}_\mu\, \partial_\tau X_2 = 0 \quad &\xleftrightarrow{T_1} \quad \partial_\tau \left(X_1 + \cot(\phi_\mu) X_2\right) = 0, \\ \partial_\sigma X_2 - \mathcal{F}_\mu\, \partial_\tau X_1 = 0 \quad &\xleftrightarrow{T_1} \quad \partial_\sigma \left(X_2 - \cot(\phi_\mu) X_1\right) = 0. \end{aligned} \tag{9}$$

This is illustrated in figure 1. The T-duality also identifies commutative and noncommutative internal geometries. In the T-dual picture the electric and magnetic quantum numbers that characterise the gauge theory on the Dp-brane map to the winding numbers of the D$(p-1)$-brane on the circles of the torus, $(n, m) \in \mathbb{Z}^2 \simeq H^1(\mathbb{T}^2, \mathbb{Z})$. In the following we shall always employ the T-dual "branes at angles" picture to illustrate the stacks of coincident D9-branes with different magnetic fluxes, which we meet in the orientifold construction.



Non-commutative:

$$
\begin{aligned}
\mathcal{F} &= \frac{2\pi m}{nR_1R_2} \\
U &= i\frac{R_2}{R_1}, \quad T = iR_1R_2
\end{aligned}
$$

Commutative:

$$
\begin{aligned}
\cot(\phi) &= \frac{2\pi m}{nR_1R_2} \\
U' &= iR_1R_2, \quad T' = i\frac{R_2}{R_1}
\end{aligned}
$$

Figure 1: T-duality of D-branes with flux

3 Type I with $\mathcal{F}$-Flux

Now we can proceed to the main object of this paper, the compactification of type I strings on a torus with magnetic flux on the D9-branes. We choose the $2d$-dimensional torus to be a product $\mathbb{T}^{2d} = \mathbb{T}^2_{(j)} \times \cdots \times \mathbb{T}^2_{(j)}$ with each individual $\mathbb{T}^2_{(j)}$ of purely imaginary complex and Kähler structure as before. The various radii we denote by $R^{(j)}_{1,2}$, the voluminа by $V^{(j)}$, and the flux on any individual stack of coincident D9_μ-branes we call $\mathcal{F}^{(j)}_\mu$. The appropriate T-duality T_1 on all the x_1 directions translates to a geometrical setting with D$(9-d)$-branes at angles $\phi^{(j)}_\mu$. It also affects the world sheet parity Ω which gets combined with a reflection $\mathcal{R}$ in all x_1 directions: $T_1\Omega T_1^{-1} = \Omega\mathcal{R}$. The standard procedure of the construction of type I compactifications as orientifolds of type IIB next requires to compute the one-loop massless tadpoles of the Klein bottle closed string amplitude and then introduce open string sectors into the theory in order to cancel the divergencies. As the models we consider will be shown to break supersymmetry, the one-loop amplitude does not vanish on the whole, in particular a NSNS tadpole survives. Also there can be tachyons in the open string spectrum, their masses depending on the concrete choice of the radii. Assuming some mechanism to stabilize their vacuum expectation values, they may serve as Higgs bosons in the effective field theory. On the other hand there are no closed string tachyons, which would signal a more serious gravitational instability.

3.1 One-loop amplitudes

First note that the presence of $\mathcal{F}$-flux only affects the open string sector, so that the Klein bottle amplitude remains unchanged. We include the result for completeness:

$$
\begin{aligned}
\mathcal{K} &= 2^{5-d} c\,(1-1) \int_0^\infty \frac{dt}{t^{6-d}}\, \mathrm{Tr}\left(\frac{\Omega}{2}\mathcal{P}_{\mathrm{GSO}} e^{-2\pi t\left(L_0+\bar{L}_0\right)}\right) \\
&= 2^{3-d} c\,(1-1) \int_0^\infty \frac{dt}{t^{6-d}} \frac{\vartheta\begin{bmatrix}0\\1/2\end{bmatrix}^4}{\eta^{12}} \prod_{j=1}^{d}\left(\sum_{r,s\in\mathbb{Z}^2} e^{-\pi t\left(r^2/R_1^{(j)\,2} + s^2/R_2^{(j)\,2}\right)}\right) \qquad (10)
\end{aligned}
$$

with $c = V_{10-2d}/\left(8\pi^2\right)^{5-d}$ and $\alpha' = 1$. Its contribution to the RR tadpole is to be found by a modular transformation into the tree channel:

$$\tilde{\mathcal{K}} \sim \int_0^\infty dl\ 2^{13-d} \prod_{j=1}^{d} V^{(j)}. \tag{11}$$

We now add D9$_\mu$-branes into the background, at least some with $\cot(\phi_\mu^{(j)}) = \mathcal{F}_\mu^{(j)} \neq 0$, and compute the open string diagrams. The symmetry under Ω also forces to introduce the D9-branes pairwise, any $(n_\mu^{(j)}, m_\mu^{(j)})$ accompanied by its image $(n_{\mu'}^{(j)}, m_{\mu'}^{(j)}) \equiv (n_\mu^{(j)}, -m_\mu^{(j)})$. For two types μ and ν of D9-branes then at least four kinds of open strings have to be regarded, which is depicted in figure 3:

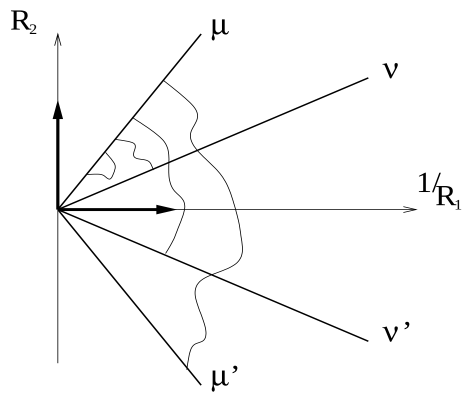


Figure 2: Open string sectors

The entire annulus amplitude can then be decomposed into sectors as follows:

$$\begin{aligned}\mathcal{A} &= c \int_0^\infty \frac{dt}{t^{6-d}}\ \mathrm{Tr}_{\mathrm{open}} \left(\frac{1}{2} \mathcal{P}_{\mathrm{GSO}} e^{-2\pi t L_0} \right) \\ &= \sum_\mu \left(\mathcal{A}_{\mu\mu} + \mathcal{A}_{\mu\mu'} + (\mu \leftrightarrow \mu') \right) + \\ &\quad \sum_{\mu<\nu} \left(\mathcal{A}_{\mu\nu} + \mathcal{A}_{\mu\nu'} + \mathcal{A}_{\mu'\nu} + \mathcal{A}_{\mu'\nu'} + (\mu, \mu' \leftrightarrow \nu, \nu') \right),\end{aligned} \tag{12}$$

where each term has to be weighted by the intersection number $I_{\mu\nu}$ of the two branes in question. By general topological arguments this is

$$I_{\mu\nu} = \prod_{j=1}^{d} \left(m_\mu^{(j)} n_\nu^{(j)} - n_\mu^{(j)} m_\nu^{(j)} \right). \tag{13}$$

The result finally reads

$$\begin{aligned}\mathcal{A}_{\mu\nu} &= c\ 2^{-2}\ \mathrm{N}_\mu \mathrm{N}_\nu I_{\mu\nu} \int_0^\infty \frac{dt}{t^{6-d}} \sum_{\alpha,\beta\in\{0,1/2\}} (-1)^{2(\alpha+\beta)} e^{2i\alpha \sum_j (\phi_\nu^{(j)} - \phi_\mu^{(j)})} e^{i\pi d/2} \\ &\qquad \times \frac{\vartheta \left[{-\beta \atop \alpha} \right]^{4-d} \prod_{j=1}^d \vartheta \left[{-(\phi_\nu^{(j)} - \phi_\mu^{(j)})/\pi - \beta \atop \alpha} \right]}{\eta^{12-3d} \prod_{j=1}^d \vartheta \left[{-(\phi_\nu^{(j)} - \phi_\mu^{(j)})/\pi - 1/2 \atop 1/2} \right]}\end{aligned} \tag{14}$$

leading to the following contribution to the massless RR tadpole

$$\tilde{\mathcal{A}}_{\mu\nu} \sim \int_0^\infty dl\ 2^{3-d}\ \mathrm{N}_\mu \mathrm{N}_\nu \prod_{j=1}^{d} \left(\frac{m_\mu^{(j)} m_\nu^{(j)}}{V^{(j)}} + n_\mu^{(j)} n_\nu^{(j)} V^{(j)} \right). \tag{15}$$

Interestingly, the existence of terms proportional to the volume of some the $\mathbb{T}^2$, signals that D9-branes with additional gauge flux not only carry the charge of a conventional D9-brane but also charges of D$(9-2i)$-branes, $i = 1...d$. Further, we again meet the electric quantum numbers n_μ rescaling the charges: A D9-brane wrapping some $\mathbb{T}^2$ twice just carries twice as much charge. This fact will be shown to enable a very simple method to lower the rank of the resulting gauge group.

The final contribution comes from the Möbius strip, resulting only from open strings invariant under Ω, i.e. stretching between some D9_μ-brane and its image D$9_{\mu'}$-brane:

$$\mathcal{M} = c \int_0^\infty \frac{dt}{t^{6-d}}\ \mathrm{Tr}_{\mathrm{open}} \left(\frac{\Omega}{2} \mathcal{P}_{\mathrm{GSO}} e^{-2\pi t L_0} \right) = \sum_\mu (\mathcal{M}_{\mu\mu'} + \mathcal{M}_{\mu'\mu}). \tag{16}$$

Now the individual sectors have to be weighted with the number $I_{\mu\mu'}^{(\Omega)}$ of invariant intersections only. The result is

$$\mathcal{M}_{\mu\mu'} = c\ 2^5\ \mathrm{N}_\mu I_{\mu\mu'}^{(\Omega)} (-1)^d \int_0^\infty dl \sum_{\alpha,\beta\in\{0,1/2\}} (-1)^{2(\alpha+\beta)} e^{4i\alpha\sum_j \phi_\mu^{(j)}} \frac{\vartheta\left[{-\beta \atop \alpha}\right]^{4-d} \prod_{j=1}^{d} \vartheta\left[{-2\phi_\mu^{(j)}/\pi-\beta \atop \alpha}\right]}{\eta^{12-3d} \prod_{j=1}^{d} \vartheta\left[{-2\phi_\mu^{(j)}/\pi-1/2 \atop 1/2}\right]}$$

and its contribution to the massless RR tadpole:

$$\tilde{\mathcal{M}}_{\mu\mu'} \sim \int_0^\infty dl\ 2^{9-d}\ \mathrm{N}_\mu \prod_{j=1}^{d} n_\mu^{(j)} V^{(j)}. \tag{17}$$

The amplitudes combine into the perfect square and the cancellation in, for instance, six dimensions ($d = 2$) requires:

$$\begin{aligned} &\text{D9-brane charge,} \quad V^{(1)}V^{(2)} : \sum_\mu \mathrm{N}_\mu n_\mu^{(1)} n_\mu^{(2)} = 16, \\ &\text{D5-brane charge,} \quad \frac{1}{V^{(1)}V^{(2)}} : \sum_\mu \mathrm{N}_\mu m_\mu^{(1)} m_\mu^{(2)} = 0. \end{aligned} \tag{18}$$

The charges which would correspond to D7-branes cancel by the Ω symmetry of the D-brane setting. In four dimensions with $d = 3$ all three kinds of D5-brane charges, identified by one volume factor in the numerator and two in the denominator, have to be cancelled, while the D7- and D3-brane charges again vanish automatically. Pure D9-branes with vanishing magnetic flux enter the tadpole cancellation with $m_\mu = 0$ and D5-branes corresponding to infinite flux with $n_\mu = 0$. We have also calculated the conditions for a tadpole cancellation in the $\mathbb{Z}_2$ orbifold, where in addition to the D9-brane charge as well a net D5-brane charge of 16 is required. This opens additional options for the construction of interesting models, such as vacua entirely without D5-branes. It even appears to be possible to maintain supersymmetry [4].

3.2 Gauge group, chirality and supersymmetry

As a D9-brane with $\mathcal{F} \neq 0$ is not invariant under Ω, but maps to the brane of opposite flux, there is no Ω projection in its open string spectrum. The resulting gauge group on a stack of N such branes is therefore $U(\mathrm{N})$ instead of $SO(\mathrm{N})$ or $Sp(\mathrm{N})$. One can then directly realize gauge groups of the type

$$SO(\mathrm{N}_9) \times Sp(\mathrm{N}_5) \times \prod_{\mu} U(\mathrm{N}_\mu), \tag{19}$$

and has also got the possibility to lower the rank simply by choosing any "electric" winding number $n_\mu^{(j)} > 1$. The fourdimensional spectrum of massless fermions is generic, independent of any radii, and chiral in any open string sector, where two D9-branes have $\mathcal{F}_\mu^{(j)} - \mathcal{F}_\nu^{(j)} \neq 0$ for all j. The multiplicities of states are essentially given by the intersection numbers of the respective D6-branes in the T-dual picture and the general results for the spectrum have been presented in [3]. Supersymmetry is actually always broken, even if sometimes in a rather subtle fashion.

3.3 An example

In this final section we shall briefly mention an example which is meant to demonstrate the power and simplicity of a "bottom-up" stategy of model construction, which consists in only putting fluxes on type I D9-branes to engineer the desired gauge group and spectrum. The D-brane configuration shown in figure 3.3

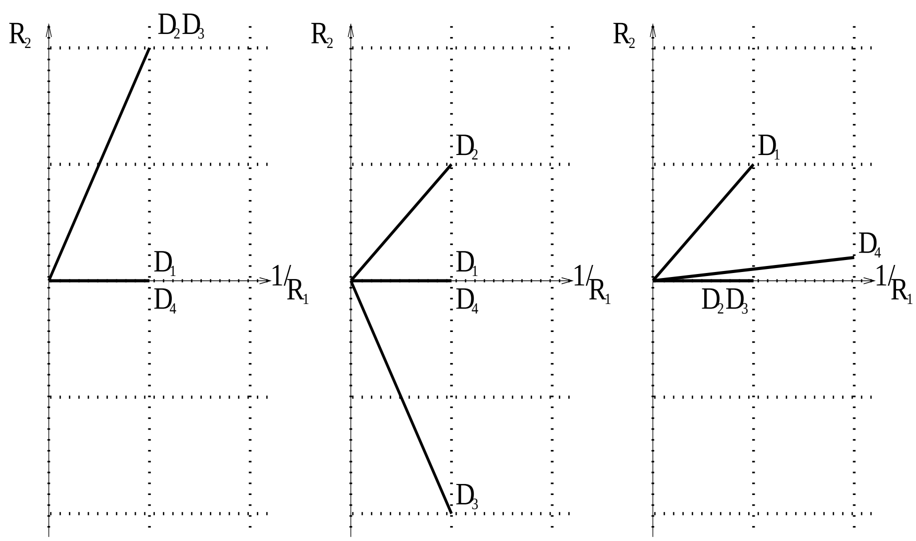


Figure 3: The four generation D-brane setting

supports a gauge group $U(3) \times U(2) \times U(1)^2$ in the effective theory. One of the four abelian factors is found to be anomalous, which is consistent with a single Green-Schwarz mechanism to be possible in order to cancel the anomaly and decouple the gauge boson. The fermion spectrum includes four generations of standard model fermions with right handed neutrinos included, and one of the nonanomalous $U(1)$ factors has suitable quantum numbers to serve as hypercharge.

Unfortunately, two major drawbacks need to be mentioned which prevent a really phenomenological model building so far: First, the number of fermion generations is always even, which has its reason in the arithmetics of intersection numbers. But even more troublesome, a "large volume" compactification is not compatible with a chiral spectrum. Any two D-branes with unequal flux on all $\mathbb{T}^2$ leave no transverse direction that could become large. Again this can be better understood in the dual picture, where any two D$(9-2d)$-branes at nonvanishing relative angles always span the entire internal space. Therefore the string scale cannot be chosen at the electroweak scale and supersymmetry breaking is obsolete.

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